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Strategic Reasoning in Resource Allocation

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Based on my work with Stefan Penczynski and Ariel Rubinstein

Resource allocation games

- R&D competition over a number of markets
- Election campaign in the US
- Cyber security (attacker-defender games)
- Spectrum auctions / oil lease auctions with limited budget

Our project

- **Goals**

  - A glimpse on the reasoning in competitive resource allocation games

  - Detect a linkage in terms of the reasoning process across these games

- **Method**

  Choice experiments with additional data:

  communication between a team of two players who play as one entity

  We analyze choices and written messages

*** We study one-shot simultaneous games**

Illustration

Colonel Blotto

- **Each player allocates 120 troops across 6 battlefields**
- In each battlefield:
  - You win and get **1 point** if you assigned more troops
  - You get **0 points** if you have an equal or a smaller number

- You play against each of the other tournament participants
- Your score is the total number of points you accumulated
- **The colonel with the highest score wins the tournament**

Our forces:

120 troops

1 2 3 4 5 6

Enemy’s forces:

120 troops

Many strategies to consider

- A strategy is an allocation of 120 troops across 6 fields
- There are around **250 million** possible strategies
- What would “classic strategic thinking” imply?
  - Forming a belief on others’ distribution of strategies
  - Best responding to the belief

An example for a prediction (belief)

In a 10 players game, suppose that a player believes:

3 players will choose 21-21-21-21-21-15

4 players will choose 60-60-0-0-0-0

2 players will choose 31-31-29-28-0-1

- Would you come up with a belief of this form?

- Can you calculate the optimal strategy given this belief?

Multi-dimensional reasoning

(Arad and Rubinstein, 2012)

- Instead of thinking about a distribution of strategies (6-component vectors) chosen by others, people think about **aspects** or **dimensions** of others' strategies
- People decide separately about each dimension of their strategy
- They combine their decisions in the various dimensions to construct a strategy

Dimension 1: Number of reinforced fronts

- The most intuitive strategy is **20-20-20-20-20-20** ("L0")
  and it provides a starting point for reasoning

- One can try to win against 20-20-20-20-20-20 ("L1")
  by reinforcing 5 battlefields, say play **24-24-24-24-24-0**

- One can try winning against 5 reinforcements ("L2")
  by reinforcing only 4, e.g. by playing **30-30-30-30-0-0**

- And so on…

Dimension 2^L: Type of assignment (ending digit) to "neglected" fronts

- Should you neglect some battlefields completely? ("L0")
- You can assign 1 troop instead, and win against people who neglected these battlefields (assigned 0) ("L1")
- You may consider assigning 2 troops to win against people who assigned 1 troop, and so on… ("L2")

Dimension 2^H: Type of assignment (ending digit) to "reinforced" fronts

- People intuitively think of multiples of ten ("L0")
- To trap a rival who deploys 30 troops, deploy 31… ("L1")
- To trap a rival who deploys 31 troops, deploy 32… ("L2")

Dimension 3: Order of divisions

- What should be the location of the strong/weak divisions?
  - **Which** battlefields should be neglected?
  - **Which** battlefields should be reinforced?
  - Increasing or decreasing order of strength of divisions?

Combining the dimensional decisions

- Suppose that a player considers all the above dimensions, performs two steps of reasoning in Dimensions 1 and 2, and focus on the middle fronts in Dimension 3

- The player may pick, for example, the strategy:

2 - 31 - 32 - 32 - 21 - 2

Back to the Start

Our research questions

(Arad and Penczynski, 2022)

- Do people **actually** think in terms of dimensions?
- If so, what are these dimensions?
- Which dimensional decision rules are used? (not today)
- Is multi-dimensional reasoning relevant to additional interactions?
- Could we identify common dimensions in a class of games?

Experimental Design

Participants and procedure

- **Participant:** 250 students
- Each played a number of resource allocation games in our lab:

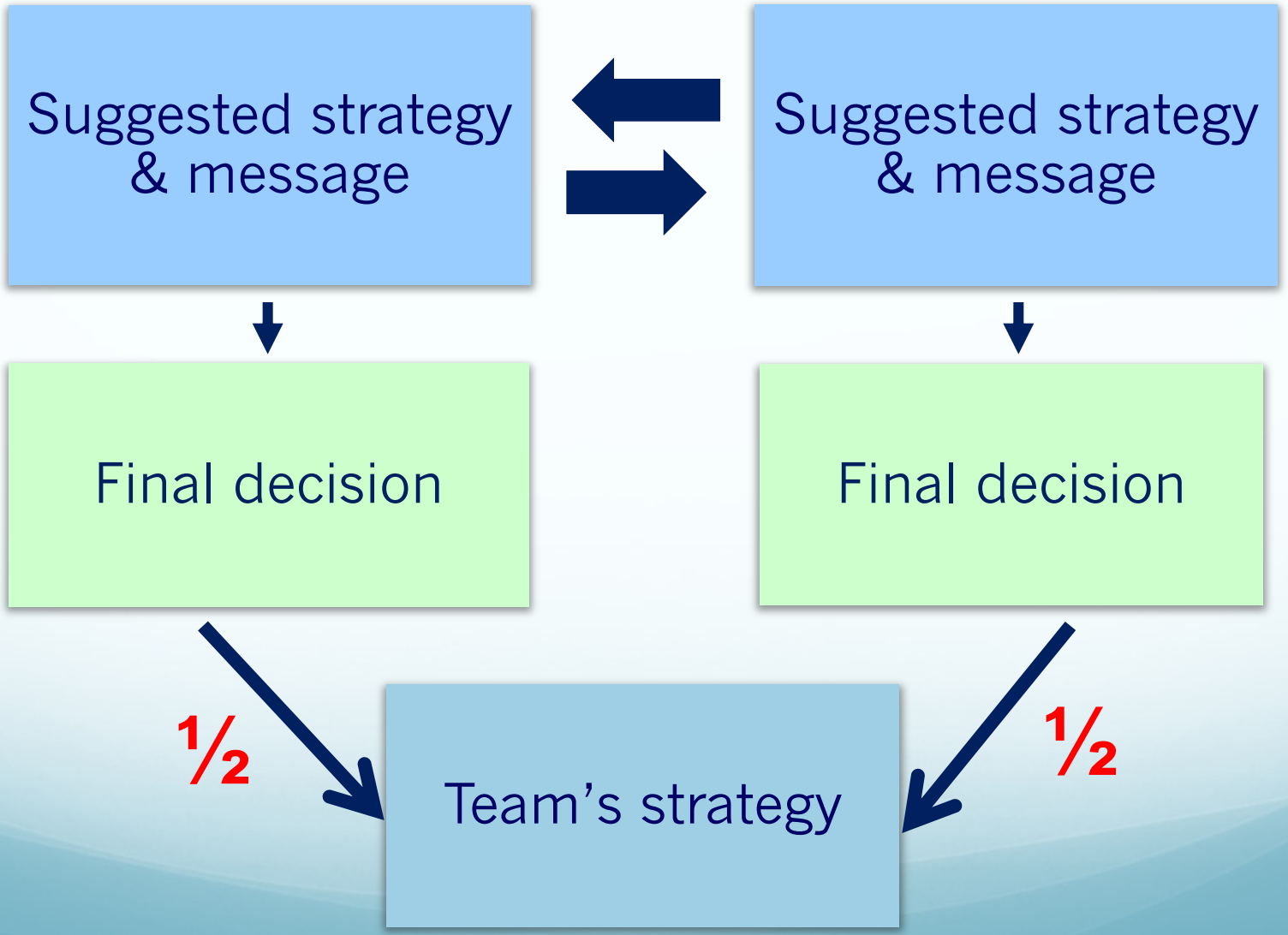
- Colonel Blotto game

- First-price multi-object auction

- All-pay multi-object auction

- **Compensation:** according to performance in the games
- A team of two anonymous participants **play as one entity**
- **A new teammate** is randomly assigned in each game

Communication protocol (Burchardi and Penczynski, 2014)



Message Classification

Independent classifications

- Two research assistants classified the messages in each game
- The individual reasoning in a single game (in a message) is classified as either "**multi-dimensional**" or "**other**"
- The classification is based **solely on the text**
- The classification of "multi-dimensional" messages includes:
  - **The dimensions** mentioned in the text (84% agreement rate)
  - **The dimensional decision rule** used in each dimension (not today)

Dimensions

Front = auction / battlefield

	Dimension
D1	Number of reinforced fronts
$D2^L$	Type of assignment to disregarded fronts
$D2^H$	Type of assignment to reinforced fronts
D3	Considerations of the identity of fronts (assignment order)

Example 1: Two dimensions in Blotto

Player 16 0-0-0-61-59-0

6 battlefields, 120 troops

Sent Message:

I would sent troops to only 2 fields and let the rest be zero. **D1**
Because nobody can win 3 fields
and when we win 2 fields we can get picked randomly because probabli another team will also win 2 fields but there is no chance to avoid tha
the difficulty is just to choose the right fields **D3**
that is psychology and i dont know so much about that :)
if you have a better idea i will change my plans :)

Example 2: Random assignment

Message from a pilot study in Germany:

Player 1 40-30-20-0-10

5 battlefields, 100 troops

Sent Message:

let me explain you my strategy: i have none.

i will just assign troops randomly. if you have a better suggestion i will pick it.

Results:

Different Games,

Similar Reasoning

Dimensions frequency in the first game

Dimension	Blotto (n=98)	Auctions (n=58)	All-pay auctions (n=52)
D1	87%	67%	77%
$D2^{L}$	24%	22%	12%
$D2^{H}$	22%	60%	23%
D3	43%	66%	56%

Number of dimensions in a message

- About 60% mention **2 or more dimensions** in their message
- About 30% mention only one dimension
- The number of dimensions per message is similar in all games

Benefits of communication analysis

- Confirms dimensional thinking in the Blotto game as well as in multi-object auctions with budget constraints and all-pay multi-object auctions
- Reveals the actual dimensions in players' mind
- Exposes decision rules that are commonly used
- Connects between the reasoning in different games

Thank you!